Muon Capture on Carbon*

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To check the predictions of a universal Fermi interaction, negative muons were brought to rest in a $1\frac{1}{2}$-in. filament scintillator chamber and the rate for the reaction $\mu^- + C^{12} \rightarrow B^{12} + \nu$ was measured. Ninety-seven B^{12} events were observed, giving a capture rate of $(6.6\pm0.9)\times10^3$/sec. This number is in agreement with the predictions of a universal Fermi interaction, a pion-induced pseudoscalar term with $g_P = +8g_A$, and a conserved vector current.

INTRODUCTION

THE rates for positive muon decay and neutron decay indicate that the coupling constants, g, for both processes are equal, within experimental error, to 1.4×10^{-49} erg cm^3. If g is truly a universal constant for all weak interactions, the rate for negative muon capture by nuclei can be predicted. The calculation is most straightforward when the muon capture results in a definite final state of the capturing nucleus. A case that has been studied[1] is that for

$$\mu^- + C^{12} \rightarrow B^{12} + \nu$$

since the capturing nucleus is experimentally convenient, the B^{12} final state can be detected by its beta decay back to C^{12}, and the interaction Hamiltonian can be calculated with some confidence. The nucleons involved in muon capture are coupled to pions and the resulting pionic terms in the interaction Hamiltonian add difficulty and interest to the prediction of the μ capture rate. Careful calculation and measurement of the μ capture rate gives information on the universality of the weak interaction, the validity of a conserved vector current, and the sign and magnitude of the pion-induced pseudoscalar term in the μ capture interaction.

Two of the six previous experiments[2,3] to measure the rate for

$$\mu^- + C^{12} \rightarrow B^{12} + \nu$$

disagree by more than three standard deviations with the other four experiments.[4–7] A new measurement using the filament scintillator chamber[8] was thought worth while. The filament chamber has the advantages of better spatial resolution than counters (to insure the B^{12} decay electron originated from the point at which the μ was captured) and better timing accuracy than spark or bubble chambers (so the B^{12} lifetime could be measured to insure that the electrons seen were in fact B^{12} decay electrons).

EXPERIMENTAL PROCEDURE

Negative muons from the University of Chicago synchrocyclotron were stopped in a $1\frac{1}{2}$-in-diam scintillation chamber. The scintillator material (polyvinyltoluene) is entirely hydrogen and carbon so that essentially all the stopping μ's form carbon mesonic atoms directly or by transfer from hydrogen. A muon takes about 10^{-10} sec to slow down, be captured, and cascade to a K orbit, and from there can either decay or be absorbed by the carbon nucleus. About 90% of the muons decay, about 9% are absorbed and lead to final states other than B^{12} ($B^{11}+n$, etc.), and about 1% of the captured muons lead to B^{12}. The B^{12} can be identified by its 21-msec half-life decay electron.

The filament chamber was made of 0.030-in.-diam $\times$3.5-in.-long plastic scintillator filaments. A minimum ionizing particle crossing a filament produced about 600 photons. Allowing for the acceptance angle for total internal reflection and attenuation losses in the filament, about 35 photons were able to escape from each end of the filament. One end of the chamber was coupled by three light pipes to six photomultipliers (see Fig. 1). In this way the chamber could be used as three separate scintillation counters. The other end of the chamber was butted directly against the front window of a three-stage image intensifier tube. The first photocathode of this tube had a 1.5-in. usable diameter and a conversion efficiency of 120 μA/lumen, or for scintillation light, an efficiency of 0.14 electron/photon.

Coils capable of providing a magnetic field of 60 G were placed above and below the last stage of the three-stage tube in order to deflect the image of the filament chamber across the 3-in.-diam output phosphor. Two small permanent magnets held the image deflected

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[1] The suitability of this reaction as a test of the universal Fermi interaction hypothesis was first pointed out by J. Tiomno to Godfrey (See reference 6).
[2] J. O. Burgman, J. Fischer, B. Leontic, A. Lundby, R. Meunier, J. P. Stroot, and J. D. Teja, Phys. Rev. Letters **1**, 469 (1958).
[3] H. V. Argo, F. B. Harrison, H. W. Kruse, and A. D. McGuire, Phys. Rev. **114**, 626 (1959).
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[5] E. J. Maier, B. L. Bloch, R. M. Edelstein, and R. T. Siegel, Phys. Rev. Letters **6**, 417 (1961).
[6] T. N. K. Godfrey, thesis, Princeton University, 1954 (unpublished).
[7] B. L. Bloch, thesis, Carnegie Institute of Technology, 1960 (unpublished).
[8] G. T. Reynolds, IRE Trans. Nucl. Sci., NS-7, nos. 2–3 (1960); G. T. Reynolds, R. A. Swanson, and D. B. Scarl, Rev. Sci. Instr. **31**, 1011 (1960).

1/3 diam to the left while the incoming muon track was recorded and for 2 msec thereafter to insure recording all muon decays. A current pulse through the deflection coils then moved the image 1/3 diam to the right and held it in this position for 67 msec to record B^{12} decay electrons. A fast lens imaged the light from the image tube output phosphor onto the photocathode of a one-stage intensifier orthicon. The output of the intensifier orthicon was amplified by the standard amplifiers in a RCA TK-31A television system and presented on a 5-in. monitor kinescope. The kinescope was photographed on 35-mm film. The over-all gain in light intensity from filament chamber to film was about 50 000.

The capture rate to B^{12} was compared with the free-decay rate of the muon by comparing the number of photographs which showed B^{12} decay electrons with the number of photographs which showed muon decay electrons in a calibration experiment.

The time, in msec, between the photomultiplier signal from the muon coming to rest in the scintillation chamber and the signal from the B^{12} decay electron was recorded on the film. Comparison of the distribution of these times with the known B^{12} half-life acted as a check that true B^{12} decays were being seen.

164 MeV/c muons from pion decays in flight were degraded in energy by 1.5 in. of copper so that they stopped in the center of the filament chamber. The contamination from stopping pions was less than 2%. The counters, collimator, and absorber were arranged as in Fig. 1. S_1 and S_2 were 10-×10-×1/4-in. plastic scintillation counters. S_0 was a 1-×1-×1/4-in. scintillation counter. S_3, S_4, and S_5 were the filament chamber sections which were used as three separate counters. Two photomultipliers viewed each chamber section.

A stopping muon was indicated by the combination

$$(S_1S_2S_0)(S_{3a}S_{3b})(S_{4a}S_{4b})(\overline{S_{5a}S_{5b}}).$$

If no decay electron indicated by $(S_{3a}S_{3b})$ or $(S_{4a}S_{4b})$ or $(S_{5a}S_{5b})$ occurred within 10 μsec the cyclotron beam was turned off for 60 msec, the television kinescope was gated on, and the clock began generating pulses at 1-msec intervals to advance a timing scaler. The timing scaler readout was a pair of Nixie tubes[9] in the field of view of the camera photographing the TV screen. The clock also advanced the channel scaler of a Nuclear Data 256-channel pulse-height analyzer. Two milliseconds after the μ stopped, the image of the chamber was deflected on the output phosphor of the three-stage image intensifier in order to record the B^{12} decay electron in a separate picture from the μ stop. When $(S_{3a}S_{3b})$ or $(S_{4a}S_{4b})$ or $(S_{5a}S_{5b})$ in anticoincidence with (S_1S_2) indicated a B^{12} decay electron, the Nixies in the field of the camera were flashed on for one-half millisecond recording the time in msec between the μ stop and B^{12} decay on the photograph of the tracks. A store pulse was also sent to the pulse-height analyzer when a B^{12} pulse occurred.

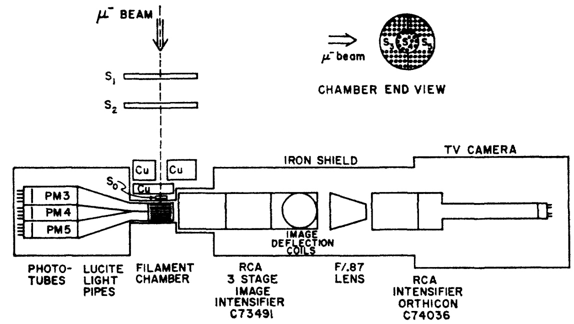

FIG. 1. The filament chamber and associated image tubes. The three-stage tube had an over-all gain of 3000 when run at a voltage of 30 kV. The intensifier stage of the orthicon had an estimated gain of 30 when run at 14 kV.

Two general types of pictures were taken. The first type gave information about the operation of the chamber, and the second type was used to measure the B^{12} capture rate. To check the chamber operation, S_0 was moved behind the chamber and 1.2 times minimum ionizing muons with a momentum of 164 MeV/c were directed through the system. One thousand pictures taken by triggering on $S_1S_2S_0$ coincidences were analyzed by dividing the image of the chamber into 10 horizontal strips and 11 vertical strips. The horizontal strip through which each track passed was recorded as were the vertical strips in which there were gaps in the track. A normalized plot of the number of gaps in each of the 110 regions into which the chamber was divided showed a uniform distribution with no apparent dead spots. The number of gaps increased slightly at the edges of the chamber, which was to be expected since the resolution and light collection ability of the optical system were worse near the edge of the image.

250 of the same type tracks were scanned by counting the number of filaments lit per track. Since counter S_0 behind the chamber was 1×1 in., only tracks passing through the chamber near the middle were photographed. The chamber was 35 filaments wide at this point, giving each track a chance to light 35 filaments. To light a filament here means that enough light was produced by the filament and image tube system to record the image of the filament on film. Since each filament through which the muon passes is recorded as either lit or not lit, the distribution in lit filaments per track will be approximately binomial. This distribution is characterized by p, the probability for success in one trial, and n, the number of trials; ($n=35$ in this case). By plotting, as in Fig. 2, the observed number of filaments lit per track and choosing p in the binomial distribution to give the best fit to the observed curve, we find $p=0.37$. This means that when a nearly minimum ionizing particle crosses a filament the light from the filament is recorded on the film only 37% of the time with the present intensifier system. The peak at

[9] Digital display tubes made by Burroughs Corporation.

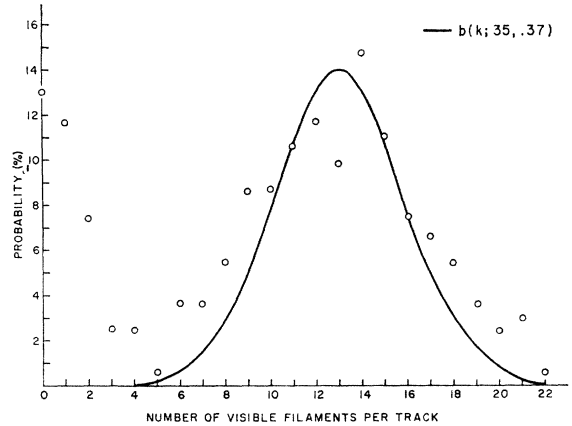

FIG. 2. A comparison of the number of chamber filaments lit per track for a particle crossing 35 filaments and the binominal distribution for 35 trials with a probability of success of 37%.

zero filaments on Fig. 2 was shown to be due to accidentals caused by the high pion flux through S_1 and S_2.

In order to measure the muon absorption rate to B^{12}, S_0 was moved in front of the chamber, copper absorber and collimators were added, and pictures were taken of muons which stopped in the center section of the chamber. 2750 pictures were taken triggering on all stopping muons in order to establish the chamber efficiency for detecting decay electrons. The deflection pulse was turned off in these calibration pictures in order that the image of the μ decay electron could remain stationary on the phosphor for 60 msec and thus simulate a B^{12} decay electron. 6823 pictures were taken of stopping muons which did not decay within 10 μsec. These pictures were used to look for B^{12} decay electrons.

DATA ANALYSIS

If R_b is the absorption rate to all states of B^{12},

$$R_b = N_b/N_s R_t,$$

where N_s is the total number of stopping muons and N_b is the number of stopping muons which lead to B^{12}. R_t, the total disappearance rate for negative muons in carbon, is given by

$$R_t = R_b + R_d,$$

where R_d is the free-muon decay rate.

To express N_b/N_s in terms of photographs and scaler readings

$$N_b/N_s = (P_{2b}/S_{2s})(1/\epsilon_{2b}).$$

P will in each case denote a type of scanned photograph, S a scaler reading. Subscript 2 denotes the B^{12} run in which only nondecaying muons were photographed; subscript 1 will denote a calibration run in which all stopping muons were photographed. Thus, P_{2b} is the number of pictures which showed B^{12} decay electrons, S_{2s} is the scaler reading for stopping muons. ϵ_{2b} is the chamber efficiency for detecting a B^{12} event:

$$\epsilon_{2b} = \epsilon_{1d}(T_b/T_d)(\epsilon_{2s}/\epsilon_{1s})(\epsilon_{2d}/\epsilon_{1d}).$$

ϵ_{1d} is the efficiency for detecting muon decay electrons, T_b and T_d are lifetime factors for the fractions of B^{12} decays and muon decays which could be detected in the time the chamber was sensitive. ϵ_{2s} is the efficiency for seeing a stop in the type 2 pictures. This is not equal to ϵ_{1s} (the efficiency for seeing a track in the type 1 pictures) since the nondeflected type 1 stops can give off light for 60 msec (phosphor glow) and the type 2 stops are deflected after 2 msec and the energy in the tail of the phosphor decay curve is swept into the deflected picture.

$$\epsilon_{1d} = P_{1d}/(S_{1s}R_d/R_t),$$

i.e., the efficiency for seeing decay electrons is the number of pictures containing decay electrons divided by the number of μ stops recorded times the fraction of stops expected to give decay electrons. ϵ_{1s} and ϵ_{2s}, the efficiencies for seeing μ stop tracks in type 1 and type 2 pictures, are given by

$$\epsilon_{1s} = P_{1s}/S_{1s},$$

$$\epsilon_{2s} = P_{2s}/S_{2nd},$$

$\epsilon_{2d}/\epsilon_{1d}$ is taken to be 1, that is, decay electrons are seen with the same efficiency in B^{12} and calibration pictures. This assumption is valid due to the small size of the chamber. Almost 100% of the μ decay electrons go out of the chamber as do 75% of the boron decay electrons. Electrons which leave the chamber look identical, no matter what their energy. Of the low-energy B^{12} electrons which do not get out of the chamber all but a few percent cross more than 10 filaments.

From the above the capture rate to B^{12} is

$$R_b = R_t \frac{P_{2b}}{S_{2s}} \frac{T_d}{T_b} \frac{\epsilon_{1s}}{\epsilon_{2s}} \frac{1}{\epsilon_{1d}} = R_d \frac{P_{2b}}{S_{2s}} \frac{P_{1s}}{P_{2s}} \frac{S_{2nd}}{P_{1d}} \frac{T_d}{T_b}.$$

R_d is the free-decay rate of the negative muon assumed equal to the decay rate of the positive muon and measured by Lundy[10] to be

$$R_d = (4.539 \pm 0.005) \times 10^5 \text{ sec}^{-1}.$$

T_d was unity since the sensitive time for μ decays was very long; T_b was 0.88. The other necessary data were

$$P_{2b} = 97, \quad S_{2s} = 35819,$$
$$P_{1s} = 1411, \quad P_{2s} = 4363,$$
$$S_{2nd} = 6823, \quad P_{1d} = 412,$$

so $R_b = (4.539\times10^5)(1.648\times10^{-2}) = 7.48\times10^3$ sec^{-1}. The total statistical error was 12%, largely due to the small number of B^{12} events, P_{2b}.

Measurements by other groups[3,5] of gamma rays following μ capture have indicated that between 3%

[10] Richard A. Lundy, Phys. Rev. **125**, 1686 (1962).

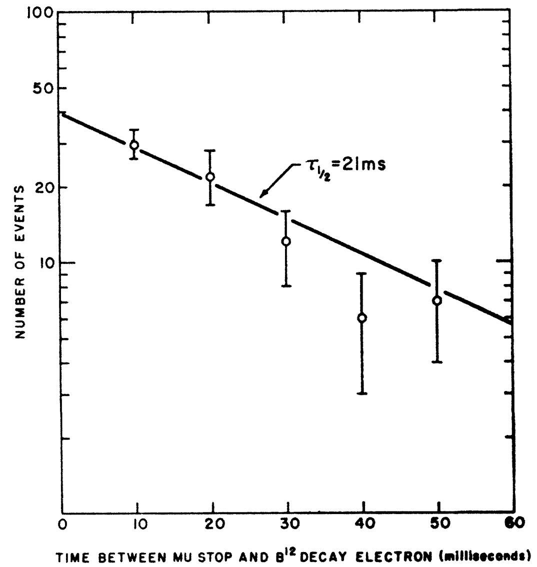

FIG. 3. Decay curve as recorded from the Nixie numbers of those photographs showing B^{12} decays. The line indicating the accepted half-life for B^{12} is in reasonable agreement with the experimental decay curve.

and 10% of μ captures leading to B^{12} give excited states of the boron nucleus. Since the quantity of interest is the capture rate to the ground state of B^{12}, the present capture rate is multiplied by 0.9 and an additional possible error of $+7\%$ is introduced.

Natural carbon contains about 1% carbon-13. If about 10% of the μ captures on carbon-12 lead to boron-11 and a neutron, one expects 10% of the captures on carbon-13 to lead to boron-12 and a neutron. Thus, this mechanism for producing boron-12 could be about 10 times more efficient than producing boron-12 from carbon-12. A possible error of -10% in the rate for formation of boron-12 is introduced by the carbon-13 contamination.

Finally, the capture rate to the ground state of B^{12} for the reaction

$$\mu^- + C^{12} \rightarrow B^{12} + \nu$$

is found to be

$$R_{\text{gnd}} = 0.9 \times 7.48 \times 10^3 = (6.7 \pm 0.9) \times 10^3 \text{ sec}^{-1},$$

with only the statistical error quoted.

TIME DISTRIBUTION

The Nixie numbers from the 97 events identified as B^{12} decays were recorded, divided into 10-msec groups starting with 6 msec and plotted in Fig. 3. A straight line drawn with the 21-msec slope of the accepted B^{12} half-life[11] fits the points well. A maximum likelihood calculation of the lifetime using the 97 B^{12} events gave a value of 15 ± 3 msec.

DISCUSSION OF RESULTS

The rate $R = (6.7\pm0.9)\times10^3$ sec^{-1} obtained with the filament scintillator chamber is to be compared with these results of previous experiments; (Table I).

As can be seen from Fig. 4, five results cluster around 6.3×10^3 sec^{-1} and two around 9.1×10^3 sec^{-1}. It has been pointed out[12] that in a pure counter measurement of the rate the result is obtained by measuring the decay curve produced by B^{12} electrons beginning some few msec after the muon has stopped. All investigators

[11] F. Ajzenberg-Selove and T. Lauritsen, Nucl. Phys. **11**, 1 (1959). A. Marques, A. J. P. L. Policarpo, and W. R. Phillips, *ibid.* **36**, 45 (1962).

[12] R. T. Siegel (private communication).

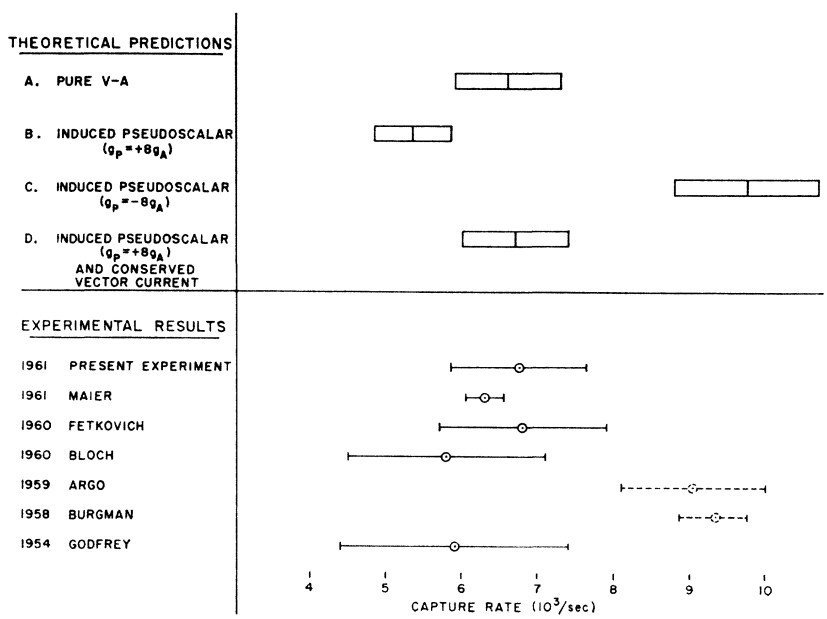

FIG. 4. Theoretical and experimental results for μ mesons captured on carbon leading to the ground state of B^{12}. Two additional sources of error as mentioned in the text are not included in the plotted experimental errors.

have found a short-lived (~3 msec) activity which confuses the first part of the 21-msec B^{12} decay curve. The latter part of the B^{12} decay curve is often difficult to separate from the constant background. Since the capture rate obtained depends on a determination of the B^{12} decay curve and its extrapolation back to zero time, large errors in rate can be generated by small errors in subtracting background or spurious activity. This is believed to be the difficulty with the two experiments which found capture rates above 9×10^3 sec^{-1}. The most recent counter experiment[5] was analyzed taking great care to make a valid extrapolation and gives a value of 6.31×10^3 sec^{-1}. The present filament chamber experiment and the bubble chamber experiment do *not* use the extrapolation technique and are much less dependent on the B^{12} decay curve. Both of these experiments agree well with 6.3×10^3 sec^{-1}.

TABLE I. Experimental capture rates.

Experiment	Source	Detector	Rate (sec^{-1})
Burgman *et al.*[a]	machine	counters	9.32±0.45
Argo *et al.*[b]	cosmic rays	counters	9.05±0.95
Fetkovich *et al.*[c]	machine	bubble chamber	6.8 ±1.1
Maier *et al.*[d]	machine	counters	6.3 ±0.24
Godfrey[e]	cosmic rays	counters	5.9 ±1.5
Bloch[f]	machine	counters	5.8 ±1.3

[a] See reference 2.
[b] See reference 3.
[c] See reference 4.
[d] See reference 5.
[e] See reference 6.
[f] See reference 7.

Recent calculations of the rate for μ capture to B^{12} have been done by Fujii and Primakoff[13] using the nuclear shell model with *jj* coupling to evaluate the nuclear wave functions, by Wolfenstein[14] using the shell model with *d*-wave neutrino emission and considering relativistic second forbidden terms, by Flamand and Ford[15] using an intermediate-coupling shell model, and by Morita and Fujii[16] using a general formalism taking into account terms of higher forbiddenness. In general, each author presents four results:

(A) The rate for a pure Fermi interaction.

(B) The rate with the addition of an induced-pseudoscalar term such that $g_P=+8g_A$.[17]

(C) The rate with the addition of an induced-pseudoscalar term such that $g_P=-8g_A$.

(D) The rate with the addition of an induced pseudoscalar term such that $g_P=+8g_A$ *and* the assumption of a conserved vector current.

[13] A. Fujii and H. Primakoff, Nuovo Cimento **12**, 327 (1959).
[14] L. Wolfenstein, Nuovo Cimento **13**, 319 (1959).
[15] G. Flamand and K. W. Ford, Phys. Rev. **116**, 1591 (1959).
[16] M. Morita and A. Fujii, Phys. Rev. **118**, 606 (1960).
[17] This relation between the coupling constants was predicted by M. L. Goldberger and S. B. Treiman, Phys. Rev. **111**, 345 (1958).

TABLE II. Predicted capture rates as calculated in references 13, 14, 15, and 16. The correction for nuclear size is taken from Flamand and Ford.

Author	Published (sec^{-1})	Corrected for nuclear size (sec^{-1})
(A)		
Wolfenstein	$7.3\ \times10^3$	$6.9\ \times10^3$
Flamand and Ford	$6.2\ \times10^3$	$6.2\ \times10^3$
Morita and Fujii	$7.0\ \times10^3$	$6.6\ \times10^3$
(B)		
Fujii and Primakoff	6.34×10^3	5.97×10^3
Wolfenstein	$5.9\ \times10^3$	$5.5\ \times10^3$
Flamand and Ford	5.28×10^3	5.28×10^3
Morita and Fujii	5.68×10^3	5.34×10^3
(C)		
Flamand and Ford	9.65×10^3	9.65×10^3
Morita and Fujii	10.39×10^3	9.77×10^3
(D)		
Fujii and Primakoff	7.86×10^3	7.39×10^3
Wolfenstein	$7.4\ \times10^3$	$6.9\ \times10^3$
Flamand and Ford	7.26×10^3	7.26×10^3
Morita and Fujii	7.12×10^3	6.69×10^3

Apart from Flamand and Ford, none of these authors take into account the effect the finite size of the C^{12} nucleus has on the muon wave function. Flamand and Ford calculate this to be a -6% correction to the capture rate. This correction leads to the compilation of theoretical results shown in Table II, where (A), (B), (C), (D) refer to the assumptions listed above.

Wolfenstein estimates his uncertainty to be $\pm20\%$, Flamand and Ford estimate $\pm10\%$. From Table II it can be seen that the variation in values between authors in any given category is of the order of $\pm5\%$.

In order to compare experimental and theoretical results, the values of R_{exp} obtained in six experiments as well as the corrected values of R_{theoret} calculated by Morita and Fujii are plotted in Fig. 4. The data are consistent with these assumptions:

(1) The weak interaction can be described as the self-interaction of a current using a single coupling constant g with a value of 1.41×10^{-49} erg cm^3.

(2) The pion-induced pseudoscalar term has a coupling constant $g_P=+8g_A$.

(3) The vector current is conserved in such a way that $g_V=g$.

The data are inconsistent with the assumption that the pseudoscalar coupling constant is $g_P=-8g_A$.

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